Recent progress in analytical aspects of kinetic equations and related fluid models

Sep 28 - 29, 2019

University of Wisconsin-Madison, Mathematics
Van Vleck Hall, Room 911

Saturday, Sept 28

08:30 - 09:30 am	Y. Guo
09:30 - 10:30 am	A. Vasseur
10:30 - 11:00 am	Coffee break
11:00 - 12:00 pm	Z. Lin
12:00 - 02:00 pm	Lunch Break
02:00 - 03:00 pm	I. Gamba
03:00 - 03:30 pm	Coffee break with Poster session
03:30 - 04:30 pm	R. Strain
04:30 - 05:30 pm	N. Pavlovic

Sunday, Sept 29

08:30 - 09:30 am	G. Bal
09:30 - 10:30 am	L. Silvestre
10:30 - 10:50 am	Coffee break
10:50 - 11:50 am	E. Carlen
11:50 - 12:50 pm	J. Bedrossian

Abstracts:

Landau equation with specular reflection boundary condition

Yan Guo

Based on a recent L^2 to L^∞ framework, we establish the global stability of Maxwellian for the Landau equation in a bounded domain with specular reflection boundary condition. This is a joint work with Hyungju Hwang, Jin Woo Jang and Zhimeng Ouyang.

Partial regularity in time for the Landau equation with Coulomb interaction

Alexis Vasseur

We consider the space homogeneous Landau equation with Coulomb potential. For this equation, Villani has proved the global existence of a special kind of weak solutions of the Cauchy problem, known as “H-solutions”, for all initial data with finite mass, energy and entropy. Whether H-solutions with smooth initial data remain smooth for all times, or blow up in finite time is one of the outstanding problems in the mathematical analysis of kinetic models. In this talk, we will show that the set of singular times for weak solutions has Hausdorff dimension at most 1/2. Surprisingly, this is the same Hausdorff dimension as for weak solutions to the 3D Navier-Stokes equation. This is a joint work with Francois Golse, Maria Gualdani, and Cyril Imbert.

Stability of some stellar models

Zhiwu Lin

First, consider gaseous stars modelled by the Euler-Poisson system. Under general assumptions on the equation of states, (with Chongchun Zeng) we proved a turning point principle that the stability of the stars is entirely determined by the mass-radius curve parametrized by the center density. In particular, the stability can only changed at points with an extremal mass. Second, (with Hadzic and Rein) we consider the stability of relativistic star clusters modeled by Vlasov-Einstein system. The above question arose in the 1960s with the discovery of quasars, which exhibit high redshift. I will discuss results which illustrate the fact that as the central redshift increases, the steady states change from being stable to being unstable. For both problems, we use a combination of first order and 2nd order Hamiltonian formulations to get the stability criterion and the semi-group estimates for the linearized equation.

Multi-component gas system models: The billiard mixing for mixtures

Irene Gamba

The lecture will focus on the mathematics of kinetic systems for mixtures of gases with different masses. This corresponds to a Boltzmann system for the evolution of vector valued probability distribution densities for non-local bi-linear collisional forms modeling ‘mixing’ of their states. The number of species is arbitrary. The interaction law, as much as the modeling of the transition probability rates, which modeled by a quantification of differential cross section for pairwise interactions, are crucial components in the dynamics. We will present recent rigorous properties developed for the multi-component gas system described by coupled Boltzmann equations corresponding to the dynamics of elastic mixing of particles characterized by their identical shapes (spheres) but different masses, both in monoatomic and polyatomic cases. We construct unique solutions on L^1 -polynomial weighted spaces with suitable weights and show propagation and generation of polynomial moments as well as exponential moments. In addition we show the solutions propagate L^p polynomial and exponential bounds, for \$1< p\le\infty\$. This work is in collaboration with Milana Pavic-Colic and Erica De La Canal.

Global mild solutions of the Landau and non-cutoff Boltzmann equation

Robert M. Strain

This paper proves the existence of small-amplitude global-in-time unique mild solutions to both the Landau equation including the Coulomb potential and the Boltzmann equation without angular cutoff. Since the well-known works (Guo, 2002) and (Gressman-Strain-2011, AMUXY-2012) on the construction of classical solutions in smooth Sobolev spaces which in particular are regular in the spatial variables, has still remained an open problem to obtain global solutions in an $L^\infty{}_{x,v}$ framework, similar to that in (Guo-2010), for the Boltzmann equation with cutoff in general bounded domains. One main difficulty arises from the interaction between the transport operator and the velocity-diffusion-type collision operator in the non-cutoff Boltzmann and Landau equations; another major difficulty is the potential formation of singularities for solutions to the boundary value problem. In this work we introduce a new function space with low regularity in the spatial variable to treat the problem in cases when the spatial domain is either a torus, or a finite channel with boundary. For the latter case, either the inflow boundary condition or the specular reflection boundary condition is considered. An important property of the function space is that the $L^\infty{}_T L^2{}_v$ norm, in velocity and time, of the distribution function is in the Wiener algebra $A(\Omega)$ in the spatial variables. Besides the construction of global solutions in these function spaces, we additionally study the large-time behavior of solutions for both hard and soft potentials, and we further justify the property of propagation of regularity of solutions in the spatial variables. To the best of our knowledge these results may be the first ones to provide an elementary understanding of the existence theories for the Landau or non-cutoff Boltzmann equations in the situation where the spatial domain has a physical boundary.
This is a joint work with Renjun Duan (The Chinese University of Hong Kong), Shuangqian Liu (Jinan University) and Shota Sakamoto (Tohoku University).

Beyond binary interactions of particles

Natasa Pavlovic

In this talk we shall discuss dynamics of systems of particles that allow interactions beyond binary, and their behavior as the number of particles goes to infinity. In particular, an example of such a system of bosons leads to a quintic nonlinear Schrodinger equation, which we rigorously derived in a joint work with Thomas Chen. An example of a system of classical particles that allows instantaneous ternary interactions leads to a new kinetic equation that can be understood as a step towards modeling a dense gas in non-equilibrium. We call this equation a ternary Boltzmann equation and we rigorously derive it in a recent work with Ioakeim Ampatzoglou.

Boundary controls in transport and diffusion equations

Guillaume Bal

Many inverse problems require that solutions of differential equations satisfy appropriate properties of linear independence, for instance the independence of gradients of solutions in the vicinity of a given point. In imaging applications, such solutions are often controlled from the boundary. For elliptic problems, the most useful theoretical tool to verify that such properties hold is based on an application of a unique continuation principle (UCP). For (phase space) transport equations, the control problem is quite different. We will consider transport problems with feasible or unfeasible boundary controls and will contrast these results with UCP.

Regularity estimates for the Boltzmann equation without cutoff

Luis Silvestre

We study the regularization effect of the inhomogeneous Boltzmann equation without cutoff. We obtain a priori estimates for all derivatives of the solution depending only on bounds of the hydrodynamic quantities: mass density, energy density and entropy density. As a consequence, a classical solution to the equation may fail to exist after certain time T only if at least one of these hydrodynamic quantities blows up. Our analysis applies to the case of moderately soft and hard potentials. We use methods that originated in the study of nonlocal elliptic equations: a weak Harnack inequality in the style of De Giorgi, and a Schauder-type estimate. We apply them to gain regularity iteratively and we combine them with a pointwise decay estimate for large velocities.

Nonequilibrium steady states for a BGK model with thermal reservoirs

Eric Carlen

We investigate kinetic models of a one-dimensional gas in contact with homogeneous thermal reservoirs at different temperatures. Nonlinear collisional interactions between particles are modeled by a so-called BGK dynamics which conserves local energy and particle density. Weighting the nonlinear BGK term with a parameter $\alpha \in [0, 1]$, and the linear interaction with the reservoirs by $(1 - \alpha)$, we prove that for some α close enough to zero, the explicit spatially uniform non-equilibrium steady state (NESS) is *unique*, and there are no spatially non-uniform NESS with a spatial density ρ belonging to L^p for any $p > 1$. We also show that for all $\alpha \in [0, 1]$, the spatially uniform NESS is *dynamically stable*, with small perturbation converging to zero exponentially fast. This is joint work with R. Esposito, J. Lebowitz, R. Marra and C. Mouhot.

Lagrangian chaos, almost-sure exponential mixing, and passive scalar turbulence in the Batchelor regime

Jacob Bedrossian

We will discuss several recent results on the mixing and dissipation of passive scalars by various models in stochastic fluid dynamics, including the 2D incompressible stochastic Navier-Stokes equations with non-degenerate white-in-time forcing. These results are proved by studying the Lagrangian flow map using the hypoelliptic theory of degenerate SPDE and by infinite dimensional extensions of ideas from random dynamical systems. We prove that the Lagrangian flow has a positive Lyapunov exponent (Lagrangian chaos) and show how this can be upgraded to almost sure exponential (universal) mixing of passive scalars at zero diffusivity and further to uniform-in-diffusivity mixing. Using this detailed analysis of the mixing, we are able to provide a mathematically rigorous derivation of the cumulative power spectrum of passive scalar turbulence in the Batchelor regime, formally derived by Batchelor in 1959. Here, Batchelor-regime passive scalar turbulence arises in the vanishing diffusivity limits of the stochastically forced passive scalar advected by stochastic Navier-Stokes velocities at fixed Reynolds number. Aside from the power spectrum, we are also able to derive several other fundamental properties of passive scalar turbulence in this regime, such as quantitative ergodicity estimates and anomalous dissipation in the inviscid limit. This talk is based on several joint works with collaborators Alex Blumenthal and Sam Punshon-Smith.